Small Extra Dimensions and Enhanced Symmetries in Orbifold Compactifications

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`arXiv:0803.4501`

`arXiv:0810.xxxx`

Hamburg
October 1, 2008

Motivation

- Higher dimensional setups provide promising frameworks for supersymmetric GUTs
- Size of the extra dimensions undetermined
  $\Rightarrow$ Moduli problem
- Casimir energy induces a nontrivial potential
- Compactification often leads to additional $U(1)$ symmetries
- Combination of Casimir energy with gauge symmetry breaking can lead to small extra dimensions

Outline

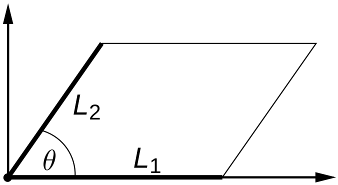
1. Example: An Orbifold GUT Model
2. Casimir Energy
3. Stabilisation

Orbifold Compactification

- Starting point: higher-dimensional setup

Here: two extra dimensions, compactified on a torus

- Torus specified by the volume $\mathcal{A}$ and shape τ



$$\mathcal{A} = L_1 L_2 \sin\theta$$
$$\tau = L_2/L_1 e^{i\theta}$$

Orbifold Compactification

- Starting point: higher-dimensional setup

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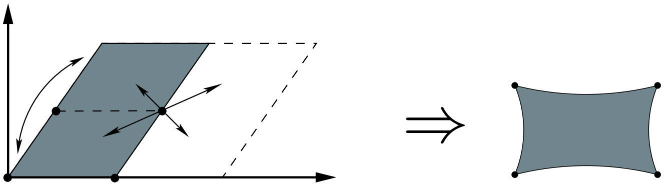
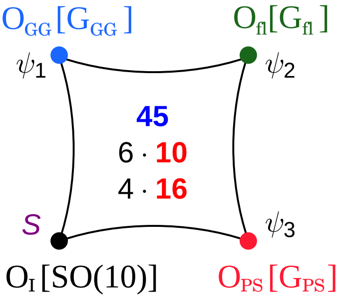
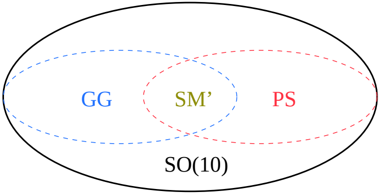


- Impose symmetry $\mathbb{Z}_2$: $y \to -y$
- Values for τ, $\mathcal{A}$?
- Casimir energy of bulk fields induces nontrivial potential
- Supersymmetry $\Rightarrow$ vanishing Casimir energy $\Rightarrow$ SUSY breaking

Gaugino Mediation in a 6D Orbifold GUT Model

Asaka, Buchmüller, Covi, Phys. Lett. **B563** (2003)





Gaugino Mediation

Kaplan, Kribs, Schmaltz, Phys. Rev. **D62** (2000)

Chacko, Luty, Nelson, Ponton, JHEP **01** (2000)

In general: soft masses for all bulk fields

Gaugino masses: $m_g = \frac{\lambda\mu}{\Lambda^2\mathcal{A}}$ Scalar masses: $m_H^2 = -\frac{\lambda'\mu^2}{\Lambda^2\mathcal{A}}$

Casimir Energy

- Consider one-loop Casimir energy of a real scalar field
- Geometry: $T^2/\mathbb{Z}_2^3$
- Fields can be either even or odd wrt a $\mathbb{Z}_2$ symmetry
- Only fields which couple to SUSY breaking brane contribute
- Boundary conditions encoded in $\alpha, \beta \in \{0, 1/2\}$

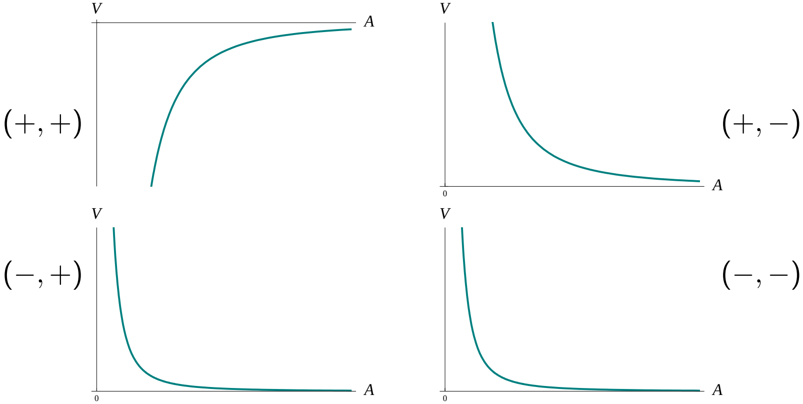
$\Rightarrow$ Four different contributions,

$$V_M^{\alpha,\beta} = \frac{1}{2}\left[\sum\right]_{m,n}^{(\alpha,\beta)} \int \frac{d^4 k_E}{(2\pi)^4} \log\left(k_E^2 + \mathcal{M}_{m,n}^2 + M^2\right)$$

$$\mathcal{M}_{m,n}^2 = \frac{4(2\pi)^2}{\mathcal{A}\tau_2} |n + \beta - \tau(m + \alpha)|^2$$

- Zeta function regularisation

Casimir Energy - Volume



- Sign and strength of Casimir force depends on boundary conditions
- General potential: $a\,V^{(+,+)} + b\,V^{(+,-)} + c\,V^{(-,+)} + d\,V^{(-,-)}$

Casimir Energy

- Analytical behaviour for small volume with τ_1 and τ_2 in the minimum:

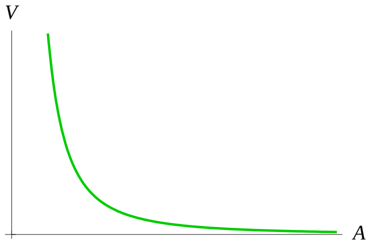
$$V_M^{(0,0)}(\tau_1 = \tfrac{1}{2}, \tau_2 = \tfrac{1}{2}, \mathcal{A}) \simeq -\frac{4\pi^3}{945\mathcal{A}^2} + \frac{\pi M^2}{360\mathcal{A}} + \mathcal{O}(M^4)$$

- Contributions for bosons and fermions come with opposite sign
$\Rightarrow$ Leading term cancels within supermultiplet
- $M^2 = M^2_{\text{SUSY}} + m^2_{\text{soft}}$
- Leading term in supermultiplet $\propto m^2_{\text{soft}}$

Casimir Energy in the Orbifold GUT Model

- Resulting potential in considered model

$$V_H \simeq -\frac{\pi}{36}\frac{\mu^2\lambda'}{\Lambda^2\mathcal{A}^2}$$



$\Rightarrow$ Can achieve repulsive force at short distances
But: Need additional ingredient for stabilisation

Breaking of $U(1)_X$

- 4D gauge symmetry: $G_{SM'} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_X$
- Vev $\langle\Phi\rangle$ breaks the additional $U(1)_X$
  $\Rightarrow$ Bulk mass $M \sim g_6 \langle\Phi\rangle$
- Quantum corrections generically induce Fayet-Iliopoulos terms at the fixed points
  Lee, Nilles, Zucker, Nucl.Phys.B **680** (2004)
  Buchmüller, Lüdeling, Schmidt, JHEP **0709** (2007)
- Localised FI terms can induce vev for bulk fields in turn
- D-flatness implies $\mathcal{A}\langle\Phi\rangle^2 \sim C\,\Lambda^2$, $C \ll 1$

Stabilisation

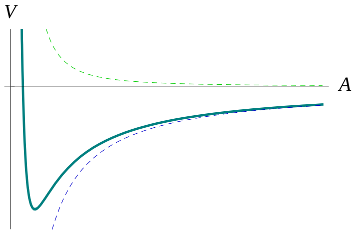
- Classical contribution to the vacuum energy density

$$V^{(0)} = -\frac{\lambda''}{\Lambda^4} \int \mathrm{d}^4\theta \, \langle S^\dagger S \; \Phi^\dagger \Phi \rangle$$
$$= -\lambda'' \frac{\mu^2 C}{\mathcal{A}}$$

- attractive for $\lambda'' > 0$
- Combine with the repulsive Casimir energy

$$V_{\text{tot}} = V^{(0)} + V^{(1)} = -\frac{\pi}{36} \frac{\mu^2 \lambda'}{\Lambda^2 \mathcal{A}^2} - \frac{\lambda'' \mu^2 C}{\mathcal{A}}$$

Volume Stabilisation

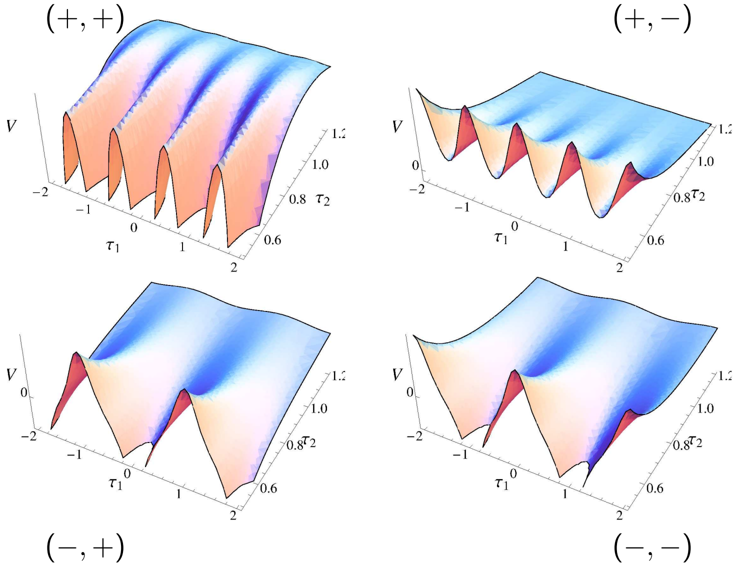


- Stable minimum at

$$\mathcal{A}_{\text{min}} = -\frac{\pi\lambda'}{36\lambda''}\frac{1}{M^2} \lesssim \frac{1}{M^2}$$

- Independent of supersymmetry breaking scale μ^2
- Cosmological constant has to be tuned to zero by a brane cosmological term

Casimir Energy - Shape

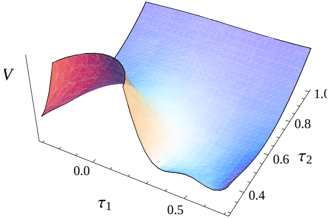


Casimir Energy - Shape

- Casimir energy invariant under modular transformations

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1$$

- Modular transformation can have fixed points which correspond to extrema in the effective potential
- In our case we have $c = 0$ mod 2 and $d = 1$ mod 2
- These transformations have a fixed point at $\tau_1 = \tau_2 = 1/2$



- Equivalent to $R_1 = \sqrt{2} R_2$ and $\theta = \pi/4 \Rightarrow$ Root lattice of SO(5)

Conclusions

- Extra dimensions can be stabilised by interplay of gauge and supersymmetry breaking
- Compactification scale naturally of $\mathcal{O}(M_{\mathrm{GUT}})$ independently of supersymmetry breaking scale μ^2
- Leads to consistent picture of Orbifold GUTs
- Shape moduli stabilised at symmetry enhanced point

Zeta function regularisation

$$V = -\frac{\mathrm{d}\zeta(s)}{\mathrm{d}s}\bigg|_{s=0}$$

where

$$\zeta(s) = \frac{1}{2}\left[\sum\right]_{m,n}\mu_r^{2s}\int\frac{\mathrm{d}^4 k_E}{(2\pi)^4}\left(k_E^2 + \frac{4}{R_z^2}\left[e^2(m+\alpha)^2 + (n+\beta)^2)\right] + M^2\right)^{-s}$$

Casimir Energy

$$
\begin{aligned}
V_M^{\alpha,\beta} = &+ \frac{M^6 \mathcal{A}}{3072\pi^3}\left[\frac{11}{12} - \log\left(\frac{M}{\mu_r}\right)\right] \\
&- \frac{M^4}{64\pi^2}\left[\frac{3}{4} - \log\left(\frac{M}{\mu_r}\right)\right]\delta_{\alpha 0}\delta_{\beta 0} \\
&- \frac{M^3 \tau_2^{3/2}}{4\pi^3 \mathcal{A}^{1/2}} \sum_{p=1}^{\infty} \frac{\cos(2\pi p \alpha)}{p^3} K_3\left(p \frac{\sqrt{\mathcal{A}} M}{2\sqrt{\tau_2}}\right) \\
&- \frac{32}{\mathcal{A}^2 \tau_2^2} \sum_{p=1}^{\infty}\sum_{m=0}^{\infty} \frac{1}{2^{\delta_{\alpha 0}\delta_{m0}}} \frac{\cos(2\pi p(\beta - (m+\alpha)\tau_1))}{p^{5/2}} \left(\tau_2^2 (m+\alpha)^2 + \frac{\mathcal{A}\tau_2 M^2}{(4\pi)^2}\right)^{\frac{5}{4}} \\
&\quad K_{5/2}\left(2\pi p \sqrt{\tau_2^2 (m+\alpha)^2 + \frac{\mathcal{A}\tau_2 M^2}{(4\pi)^2}}\right)
\end{aligned}
$$

- Dependence on regularization scale μ_r remnant of divergent bulk and brane cosmological terms