Solving the η-problem in Hybrid Inflation with Heisenberg Symmetry and Stabilized Modulas

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- Based on - arXiv:0808.2425 [hep-ph]

  with S. Antusch, M. Bastero-Gil, S.F. King, P. Kostka

Summary

- A class of SUSY hybrid inflationary model where η-problem is resolved by Heisenberg symmetry
- Associated modulas gets stabilized by large vacuum energy during inflation
- At tree level, inflaton direction is flat - protected by symmetry
- Radiative corrections lift the flatness such that $n_s < 1$ and consistent with data

Motivation

- Cosmology is now a precision science

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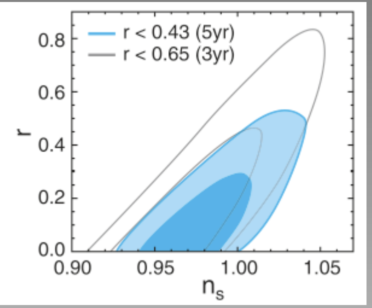
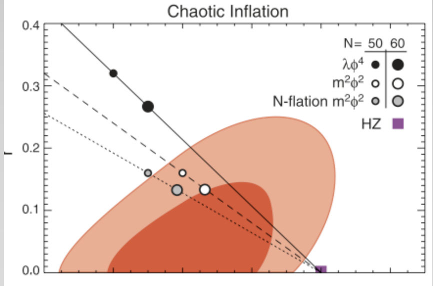
  Kolb and Turner — Dodelson — Mukhanov



- Inflation is a widely accepted paradigm for early Universe physics, but seeking for a consistent particle physics model
- Data is not precise enough and dynamics is not well understood
- Slow-Roll conditions are difficult to reconcile with known interactions of particle physics
- We present one possible toy model



η-problem

- Consider a flat inflation potential $V(\phi)$
  no reason to forbid the operator $\bar{\phi}\phi \frac{V(\phi)}{M_P^2}$
  $\eta = \left(\frac{V''}{V}\right)$ ~ 1

- In SUSY models effects of low energy SUGRA important
  flat direction (η << 1) -> extremely curved (η ~ 1, m_ϕ ~ H)

$$V(\phi) = e^{K(\phi)/M_P^2} V_0 \sim V_0(1 + \frac{\phi^2}{M_P^2} + ..) \sim V_0 + 3H^2\phi^2$$

Copeland, Liddle, Lyth, Stewart, Wands and Dine, Randall, Thomas

Moduli Problem

- Several extra scalar fields - needs to make them heavy (Dine)
- Scalar fields must find a stable minimum (Brustein, Steinhardt)
- Initial assumption: Moduli stabilization has no effect on inflationary dynamics - WRONG!
  (Brax, van de Bruck, Davis, Davis)
- Problem: Requirements for moduli and inflaton are exactly opposite!
- Avenues for solutions:
  a) specific choice of Kahler potential (Murayama et.al hep-ph/9311326)
  b) symmetry requirement of Kahler potential
  e.g shift symmetry or Heisenberg symmetry (our approach)

  (Kawasaki et.al hep-ph/0004243, Brax and Martin, Davis and Postma)

General Framework

- Hybrid Inflation - Inflaton field N and Waterfall field H

$$W = \kappa S \left(g_1(H,N) - M^2\right) + g_2(H,N)$$

$$K = (|S|^2 + |H|^2 + \kappa_S |S|^4 + \kappa_{SH} |S|^2|H|^2 + \ldots) + g_3(\rho)|S|^2 + f(\rho)$$

- Modulas is defined by $\rho = T + T^* - |N|^2$ and invariant under Heisenberg symmetry

- Main feature: $W = 0,\ W_N = W_H = W_T = 0,\ W_S \neq 0$ but $H = S = 0$

- For comparison: the standard SUSY hybrid inflation is implemented by $W = \kappa N(\bar{H}H - M^2)$

General Framework

- g_1 has to be chosen such that $|F_S|^2 = |g_1(H = 0, N) - M^2|^2$ typically it would depend on H

- g_2 leads to positive N-dependent mass^2 for H via $|F_H|^2$ during inflation - $g_2 \sim N^m H^2 \quad with \quad m \geq 1$

- g_3 together with f(ρ) shapes the potential for ρ
  $g_3(\rho)|S|^2$ term in the Kahler potential induce a contribution to the potential of the order of vacuum energy and effectively stabilize ρ

Explicit Example

- For rest of the discussion: $W = \kappa S \left(H^2 - M^2\right) + \frac{\lambda}{M_*} N^2 H^2$

$$K(H, S, N, T) \equiv |H|^2 + \left(1 + \kappa_S |S|^2 + \kappa_\rho \rho\right) |S|^2 + f(\rho)$$

- First term is similar to the SUSY hybrid
  but S= 0 both during and after inflation
  H = 0 during inflation and H ~ M in the end

$$\mathcal{L}_{\rm kin} = \frac{f''(\rho)}{4} n^2 \left(\partial_\mu n\right)^2 - \frac{f'(\rho)}{2} \left(\partial_\mu n\right)^2 - \frac{f''(\rho)}{2\sqrt{2}} n \, \partial_\mu n \, \partial^\mu t + \frac{f''(\rho)}{8} \left(\partial_\mu t\right)^2$$

$$\mathcal{L}_{\rm kin} = \frac{f''(\rho)}{4} \left(\partial_\mu \rho\right)^2 - \frac{f'(\rho)}{2} \left(\partial_\mu n\right)^2$$

$$V_{\rm F} = {\rm e}^K \left[K^{i\bar{j}} \mathcal{D}_i W \mathcal{D}_{\bar{j}} W^* - 3|W|^2\right]$$

Analysis

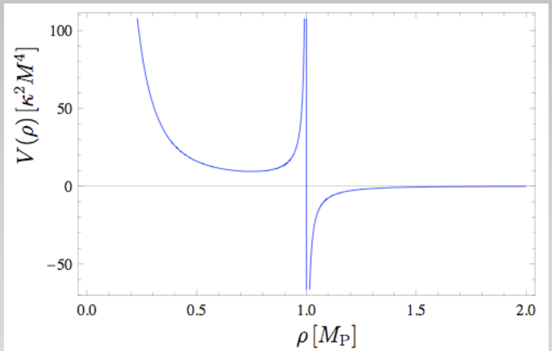
$$V_{\rm F} = e^K \left[K^{i\bar{j}} \mathcal{D}_i W \mathcal{D}_{\bar{j}} W^* - 3|W|^2\right]$$

$$V_{\rm tree} = V_{\rm F} = e^{f(\rho)} K^{-1}_{SS^*} \left|\frac{\partial W}{\partial S}\right|^2 = \kappa^2 M^4 \cdot \frac{e^{f(\rho)}}{(1+\kappa_\rho \rho)}$$ with S=H =0

- Tree level potential is exactly flat in n-direction
  Heisenberg symmetry protects n from obtaining large mass corrections from SUGRA expansion

- with $f(\rho) = -3 \ln(\rho)$

$$\rho_{\rm min} = -\frac{3}{4\kappa_\rho}$$



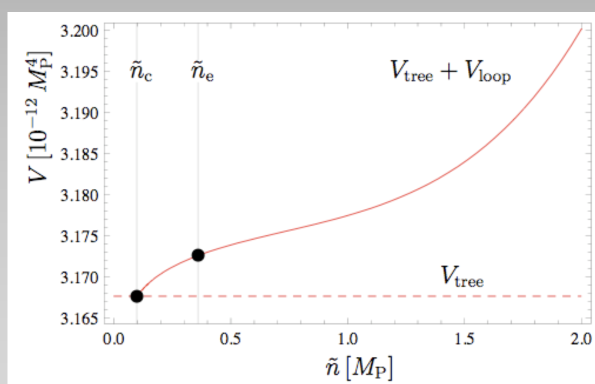
Analysis

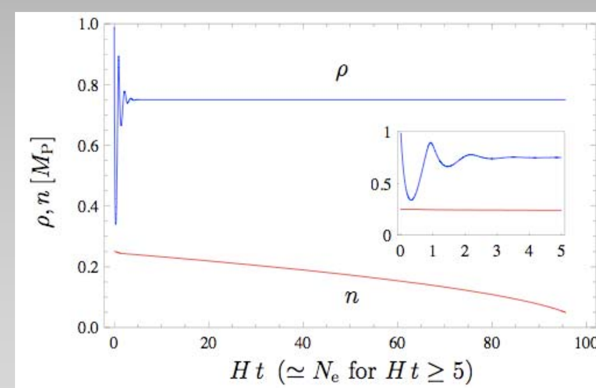
- Without assuming S=H =0, we can show that in this stable patch all the masses ρ and S mass is larger than the Hubble's constant.
- Critical value of the waterfall field: $\tilde{n}_c^2 = 8\,\frac{\kappa}{\lambda}\,(M M_*)\,\sqrt{2 - M^2}$
- Imaginary part of the fields redshifts away quickly and decouple from the real part

One-Loop Effective Potential

- N is exactly flat at the classical level
- Quantum corrections are generated by Heisenberg symmetry breaking W and broken SUSY during inflation
- Only h contributes to the n-dependent mass

$$V_{\rm eff}(n,\rho) = V_{\rm tree}(\rho) + V_{\rm loop}(n,\rho)$$





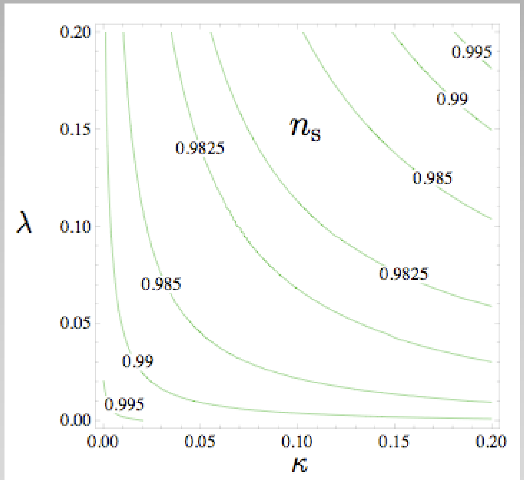
Predictions

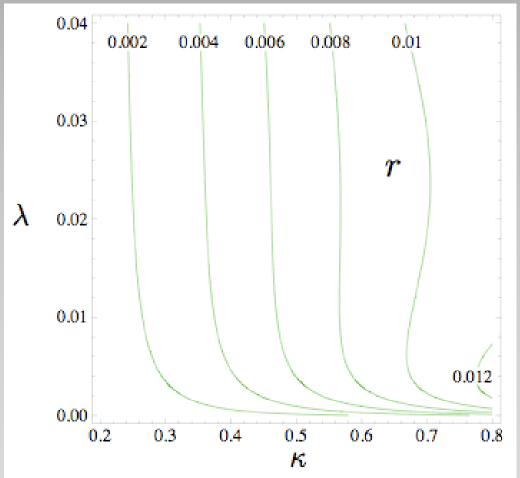
- Predictions are very similar to the standard hybrid models

  For example, $(\kappa, \lambda) = (0.05, 0.2)$

$$n_s \simeq 0.982\,, \qquad \frac{M}{M_P} \simeq 3.4 \cdot 10^{-3}$$

$$r \simeq 9.0 \cdot 10^{-5}\,, \qquad \frac{dn_s}{d\ln k} \simeq -2.4 \cdot 10^{-3}$$





Summary and Outlook

- A class of SUSY hybrid inflationary model where η-problem is resolved by Heisenberg symmetry
- Associated modulas gets stabilized by large vacuum energy during inflation
- At tree level, inflaton direction is flat - protected by symmetry
- Radiative corrections lift the flatness such that $n_s < 1$ and consistent with data
- Right handed sneutrino is an ideal candidate for inflaton in this set up
- Embedding the model in GUT framework

Thank you

$$\mathcal{L}_{\rm SUGRA} \supset -\frac{1}{2} m_{3/2} \left(G_{ij} + G_i G_j - G_{ij\bar{k}} G^{\bar{k}} \right) \chi_i \chi_j - {\rm H.c.}$$

$$(\mathcal{M}_{\rm F})_{ij} = {\rm e}^{K/2} \left(W_{ij} + K_{ij} W + K_i W_j + K_j W_i + K_i K_j W - K^{k\bar{l}} K_{ij\bar{l}} \mathcal{D}_k W \right)$$

$$(K_{i\bar{j}}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 + \kappa_\rho \rho + 4 \kappa_S |S|^2 & -\kappa_\rho N S^* & \kappa_\rho S^* \\ 0 & -\kappa_\rho N^* S & f''(\rho)|N|^2 - f'(\rho) - \kappa_\rho |S|^2 & -f''(\rho) N^* \\ 0 & \kappa_\rho S & -f''(\rho) N & f''(\rho) \end{pmatrix}$$

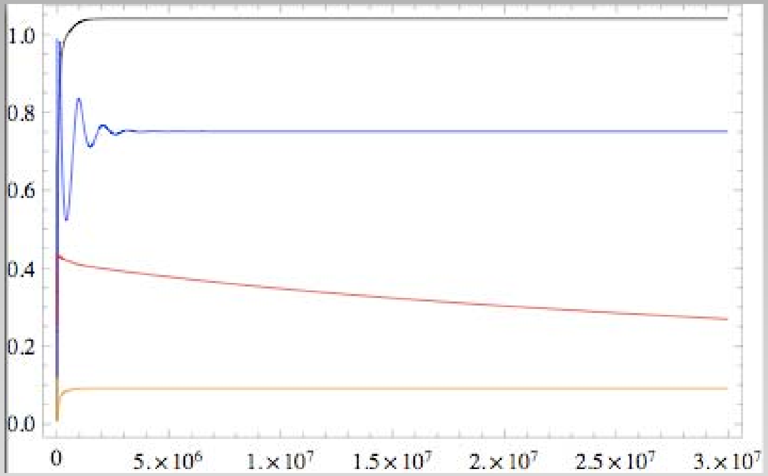
$$\ddot{h} + 3H(t)\dot{h} + \frac{\partial V}{\partial h} = 0 \,,$$

$$\ddot{n} + 3H(t)\dot{n} + \frac{f''(\rho)}{f'(\rho)} \dot{\rho}\, \dot{n} - \frac{1}{f'(\rho)} \frac{\partial V}{\partial n} = 0 \,,$$

$$\ddot{\rho} + 3H(t)\dot{\rho} + \frac{f^{(3)}(\rho)}{2 f''(\rho)} \dot{\rho}^2 + \dot{n}^2 + \frac{2}{f''(\rho)} \frac{\partial V}{\partial \rho} = 0 \,.$$

$$
\begin{aligned}
m_{\tilde{\rho}}^2 &= -\frac{16384}{81}\,\kappa_\rho^5\,\kappa^2 M^4\,,\\
m_{\tilde{s}}^2 &= \frac{4096}{27}\,\kappa_\rho^3\,\kappa_S\,\kappa^2 M^4\,,\\
m_{\tilde{h}}^2 &= -\frac{64}{27}\,\kappa_\rho^3\left[\frac{\lambda^2}{16\,M_\star^2}\,\tilde{n}^4 + 8\,\kappa^2 M^2\left(\frac{M^2}{2}-1\right)\right]\\
m_{\tilde{n}}^2 &= 0\,.
\end{aligned}
$$

$$
H^2 = \frac{1}{3}V\left(\rho_{\min}\right)\Big|_{s=h=0} = -\frac{256}{81}\,\kappa_\rho^3\,\kappa^2 M^4
$$



$$V_{\mathrm{loop}}(n) = \frac{1}{32\pi^2}\,\mathrm{Str}\left[\mathcal{M}^2(\tilde{n})\,Q^2\right] + \frac{1}{64\pi^2}\,\mathrm{Str}\left[\mathcal{M}^4(\tilde{n})\left(\ln\left(\frac{\mathcal{M}^2(\tilde{n})}{Q^2}\right) - \frac{3}{2}\right)\right]$$

$$x := \left(\frac{\lambda}{\kappa}\right)^2 \frac{1+\kappa_\rho\,\rho}{2\,(M M_*)^2}\,\tilde{n}^4$$

$$m_{\mathrm{B}}^2 = 2\,\frac{(\kappa M)^2}{(1+\kappa_\rho\,\rho)}\,\mathrm{e}^{f(\rho)}\left[x + \frac{M^2}{2} \mp 1\right]$$

$$m_{\mathrm{F}}^2 = 2\,\frac{(\kappa M)^2}{(1+\kappa_\rho\,\rho)}\,\mathrm{e}^{f(\rho)}\,x$$

$$\begin{aligned}
V_{\mathrm{loop}}(x) = \frac{(\kappa M)^4}{64\,(1+\kappa_\rho\,\rho)^2\,\pi^2}\Bigg[\\
& 4\,\mathrm{e}^{2f(\rho)}\,\left(x + M^2/2 - 1\right)^2\left[\ln\left(\frac{2\,\kappa^2 M^2\,\mathrm{e}^{f(\rho)}}{(1+\kappa_\rho\,\rho)\;Q^2}\right) + \ln\left(x + M^2/2 - 1\right) - 3/2\right] \\
& + 4\,\mathrm{e}^{2f(\rho)}\,\left(x + M^2/2 + 1\right)^2\left[\ln\left(\frac{2\,\kappa^2 M^2\,\mathrm{e}^{f(\rho)}}{(1+\kappa_\rho\,\rho)\;Q^2}\right) + \ln\left(x + M^2/2 + 1\right) - 3/2\right] \\
& - 8\,\mathrm{e}^{2f(\rho)}\,x^2\left[\ln\left(\frac{2\,\kappa^2 M^2\,\mathrm{e}^{f(\rho)}}{(1+\kappa_\rho\,\rho)\;Q^2}\right) + \ln(x) - 3/2\right]\Bigg] \\
& + \frac{(\kappa M Q)^2}{16\,(1+\kappa_\rho\,\rho)\,\pi^2}\left[\mathrm{e}^{f(\rho)}\,M^2\right].
\end{aligned}$$