Spectral Function of Majorana Neutrinos in a Thermal Bath

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Scheme

- Motivation/ Introduction
- Scalar case
- Fermion case
- (i) Diagonal Flavor Case
- (ii) Non-diagonal Flavor Case

Motivation

Non-equilibrium phenomena are responsible for some of the most interesting events in cosmology

- Reheating
- Production of Dark Matter
- Decoupling of photons (CMB)
- Baryogenesis
- ...

Introduction

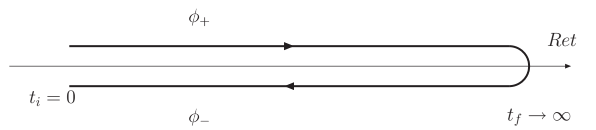
Many of non-equilibrium processes can be described via the Boltzmann equation, others requires full quantum mechanics

- Classical Boltzmann equation $\rightarrow$ Classical equation + QFT amplitudes.
- Full quantum Boltzmann $\rightarrow$ Kaddanoff-Baym + Keldysh formalism.
- Full quantum Boltzmann $\leftrightarrow$ Stochastic Langevin equation.
- Difference $\rightarrow$ non-Markovian memory effects.

The idea is to study the approach to equilibrium of particles which are in non-equilibrium with a bath.

Keldysh Formalism

- Real time formalism + Keldysh contour



- Green functions are defined in this contour

$$\Delta^>(x_1, x_2) = \langle \Phi(x_1)\Phi(x_2)\rangle = \mathrm{Tr}(\rho\Phi(x_1)\Phi(x_2)) \quad (1)$$
$$\Delta^<(x_1, x_2) = \langle \Phi(x_2)\Phi(x_1)\rangle = \mathrm{Tr}(\rho\Phi(x_2)\Phi(x_1)) \quad (2)$$

- Equilibrium: KMS condition

$$\Delta^>(\omega) = e^{\beta\omega}\Delta^<(\omega) \quad (3)$$

Scalar case

- With the definition

$$\Delta^-(x_1,x_2) = i\langle[\phi(x_1),\phi(x_2)]\rangle = i(\Delta^>(x_1,x_2)-\Delta^<(x_1,x_2)), \quad (4)$$

$$\Delta^+(x_1,x_2) = \frac{1}{2}\langle\{\phi(x_1),\phi(x_2)\}\rangle = \frac{1}{2}(\Delta^>(x_1,x_2)+\Delta^<(x_1,x_2)) \quad (5)$$

- The Kadanoff-Baym equations are given by

$$(\partial_{t_1}^2+\omega_q^2)\Delta_q^-(t_1-t_2) = -\int_{t_2}^{t_1} dt'\Pi_q^-(t_1-t')\Delta_q^-(t'-t_2) \quad (6)$$

$$(\partial_{t_1}^2+\omega_q^2)\Delta_q^+(t_1,t_2) = \int_{t_i}^{t_2} dt'\Pi_q^+(t_1-t')\Delta_q^-(t'-t_2) - \int_{t_i}^{t_1} dt'\Pi_q^-(t_1-t')\Delta_q^+(t',t_2), \quad (7)$$

- Langevin:

$$\ddot{\Psi}_q(t)+\omega_q^2\Psi_q(t)+\int_0^t dt'\Pi_q^-(t-t')\Psi_q(t') = \xi_q(t). \quad (8)$$

Solution to the first equation

- The solution for the first equation (using Laplace transform)

$$\Delta_q^-(t) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\rho_q(\omega), \tag{9}$$

where

$$\rho_q(\omega) = \frac{-2\mathrm{Im}\Pi_q^R(\omega)}{[\omega^2 - \omega_q^2 - \mathrm{Re}\Pi_q^R(\omega)]^2 + [\mathrm{Im}\Pi_q^R(\omega)]^2} \tag{10}$$

- Can be approximated by Breit-Wigner with appropriate renormalization
- The solution for the second equation

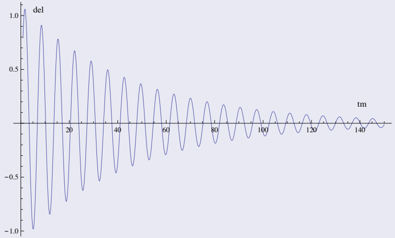
$$\Delta_q^+(t_1, t_2) \propto \int_0^{t_1} dt' \int_0^{t_2} dt'' \Delta_q^-(t_1 - t')\Delta_q^-(t'' - t_2)\Pi_q^+(t' - t''). \tag{11}$$

- Approach to equilibrium

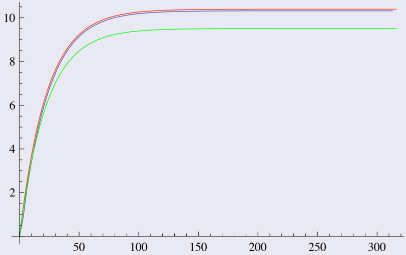
$$\Delta_q^+(\infty, \infty) = \frac{\coth\left(\frac{\beta W_q}{2}\right)}{2\sqrt{W_q^2 + \left(\frac{\mathrm{Im}\Pi_q^R}{W_q}\right)}}. \tag{12}$$

Solution to the second equation

- Spectral function vs time



- Number density vs time



Results for scalar case

Some conclusions from the scalar case

- Kadannoff-Baym can be solve exactly for this case.
- Approach to equilibrium.
- Comparisson between Boltzmann and Exact Solution.
- Memory integral does not depend on initial conditions.

Majorana Case

- Let's apply this to a Majorana case.

$$C(i\gamma^0 \frac{\partial}{\partial t_1} - \mathbf{q} - M)G^-_{q,\alpha\beta}(t_1 - t_2) = \int_{t_1}^{t_2} dt' \Sigma^-_{q,\alpha\rho}(t_1 - t')G^-_{q,\rho\beta}(t' - t_2),$$

$$C(i\gamma^0 \frac{\partial}{\partial t_1} - \mathbf{q} - M)G^+_{q,\alpha\beta}(t_1, t_2) = \int_0^{t_2} dt' \Sigma^+_{q,\alpha\rho}(t_1 - t')G^-_{q,\rho\beta}(t' - t_2) - \int_0^{t_1} dt' \Sigma^-_{q,\alpha\rho}(t_1 - t')G^+_{q,\rho\beta}(t', t_2).$$

where C is the charge conjugation matrix and Σ the self energy

Solution to the first equation

- The solution for the first equation (using Laplace transform)

$$G_q^-(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{e^{-i\omega t}}{\not{q} - M + \tilde{\Sigma}(-i\omega + \epsilon)} - \frac{e^{-i\omega t}}{\not{q} - M + \tilde{\Sigma}(-i\omega - \epsilon)}\right) d\omega, \quad (13)$$

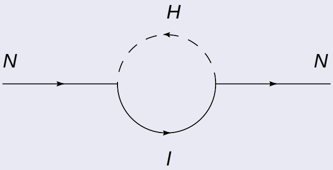
where $t = t_1 - t_2$ and

$$\tilde{\Sigma}(-i\omega \pm \epsilon) = \Sigma^R(\omega \pm i\epsilon) = \mathrm{Re}\Sigma^R \mp i\mathrm{Im}\Sigma^R. \quad (14)$$

- The biggest problem is the inversion of the above equation.
- There are gamma matrices and flavor matrices.
- Dependance on the structure of the self-energy Σ.

Structure of the self energy

- We consider a gas of strong coupled Higgs particles and leptons
- The self energy of the neutrino



- The shape of the self energy will be given by

$$\Sigma^R_{\mathbf{q}}(\omega) = (a_L \not{q} + b_L \not{u}) P_L + (a_R \not{q} + b_R \not{u}) P_R, \tag{15}$$

- $b_{R,L}$ is purely thermal
- $a_{R,L}$ and $b_{R,L}$ are complex and have flavor indices
- The real part and imaginary part are related via the Kramers-Kronig relation

Diagonal self energy

- Let's consider a diagonal 2×2 flavor, with a mass matrix $M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$ and $a_R = a_L = a$ (same for b)
- Dispersion Relation

$$\omega_{\pm} = -\frac{b_{\pm}}{(1+a_{\pm})} \pm \sqrt{\mathbf{q}^2 + \frac{M^2}{(1+a_{\pm})^2}} \tag{16}$$

- We obtain

$$\rho_{diag}(\omega) = \rho_S + \rho_q q_\mu \gamma^\mu + \rho_u u_\mu \gamma^\mu \tag{17}$$

- After some approximation

$$\rho_s \simeq -\frac{4iM}{D} q \cdot u \mathrm{Im}\,(b(\omega)), \tag{18}$$

$$\rho_q \simeq -\frac{4i}{D} q \cdot u \mathrm{Im}\,(b(\omega)), \tag{19}$$

$$\rho_u \simeq \frac{2i}{D} (\omega^2 - \omega_{\mathbf{q}}^2) \mathrm{Im}\,(b(\omega)). \tag{20}$$

Diagonal self energy

- The result in time coordinate

$$G_s^{i-}(t) = \frac{M_i}{\omega_{qi}} e^{-K_{ii}\mathrm{Im}\,(\tilde{b})t} \sin(\omega_{qi}t),$$

$$G_q^{i-}(t) = i\frac{e^{-K_{ii}\mathrm{Im}\,(\tilde{b})t}}{\omega_{qi}} \left(\omega_{qi}\gamma_0 \cos(\omega_{qi}t) + iq_i\gamma_i \sin(\omega_{qi}t)\right),$$

$$G_u^{i-}(t) = -i\frac{e^{-K_{ii}\mathrm{Im}\,(\tilde{b})t}}{\omega_{qi}^2} \left(2K_{ii}\mathrm{Im}\,(\tilde{b})\omega_{qi} \sin(\omega_{qi}t) - K_{ii}^2\mathrm{Im}\,(\tilde{b})^2 \cos(\omega_{qi}t)\right) \gamma_0.$$

where $\mathrm{Im}\,(b) = K_{ii}\mathrm{Im}\,(\tilde{b})$ and $K_{ij} = \lambda_{ik}\lambda_{kj}^*$

Non-diagonal self energy

- Let's consider now a non-diagonal 2×2 Yukawa matrix in the self energy

$$K_{ij} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \tag{21}$$

- In this scenario an exact general result can be obtained
- Diagonals and non-diagonal terms will have different poles i.e. dispersion relations will depend on the non-diagonal yukawa.
- For computation purposes can be obtain if we take $(K_{12}K_{21})/K_{ii}^2 \ll 1$
- With this one can obtain the same result as before for the diagonal part and for the nondiagonal

$$\rho_{12}(\omega) = -\frac{1}{D_1 D_2}\left[\not{q} + M_1\right]\not{u}\left[(b_{12L}\not{q} + b_{12R}M_2)P_R + (b_{12R}\not{q} + b_{12L}M_2)P_L\right]. \tag{22}$$

Conclusions

- A complete quantum analysis can be performed
- Spectral representations of majorana neutrinos can be obtain from the solution of the first Kadanoff-Baym equations
- A non-diagonal case can be treated.
- Extend this mechanism to more complicated problems.